Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2221 – 7214

20 pages

Please **do not** write on this page.

Answers written on this page
will not be marked.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

The amount, in milligrams, of a medicinal drug in the body t hours after it was injected is given by $D(t) = 23(0.85)^t,\ t \geq 0$. Before this injection, the amount of the drug in the body was zero.

(a) Write down

(i) the initial dose of the drug.

(ii) the percentage of the drug that leaves the body each hour. [3]

(b) Calculate the amount of the drug remaining in the body 10 hours after the injection. [2]

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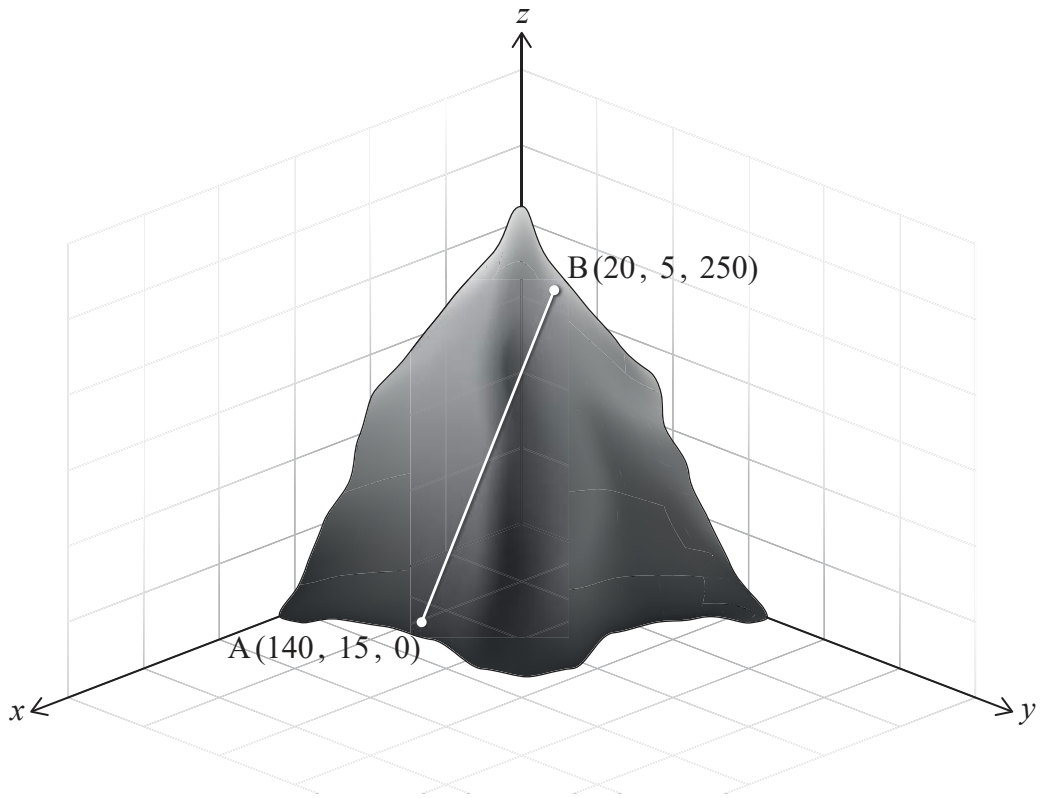
2. [Maximum mark: 5]

An inclined railway travels along a straight track on a steep hill, as shown in the diagram.

diagram not to scale



The locations of the stations on the railway can be described by coordinates in reference to x, y, and z-axes, where the x and y axes are in the horizontal plane and the z-axis is vertical.

The ground level station A has coordinates $(140, 15, 0)$ and station B, located near the top of the hill, has coordinates $(20, 5, 250)$. All coordinates are given in metres.

(a) Find the distance between stations A and B. [2]

Station M is to be built halfway between stations A and B.

(b) Find the coordinates of station M. [2]

(c) Write down the height of station M, in metres, above the ground. [1]

(This question continues on the following page)

(Question 2 continued)

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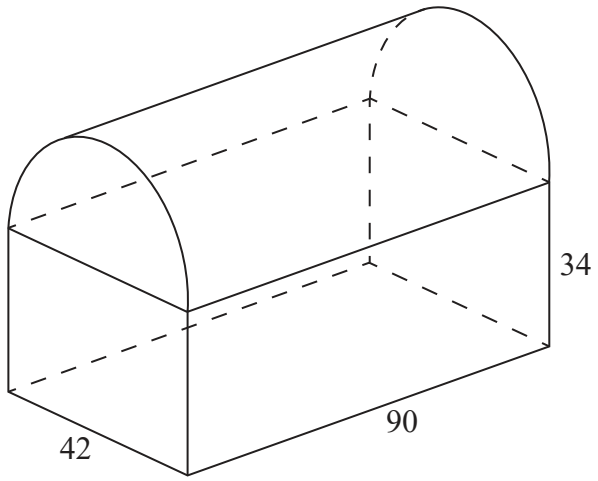
3. [Maximum mark: 7]

A storage container consists of a box of length $90\,\text{cm}$, width $42\,\text{cm}$ and height $34\,\text{cm}$, and a lid in the shape of a half-cylinder, as shown in the diagram. The lid fits the top of the box exactly. The total exterior surface of the storage container is to be painted.

Find the area to be painted.

diagram not to scale



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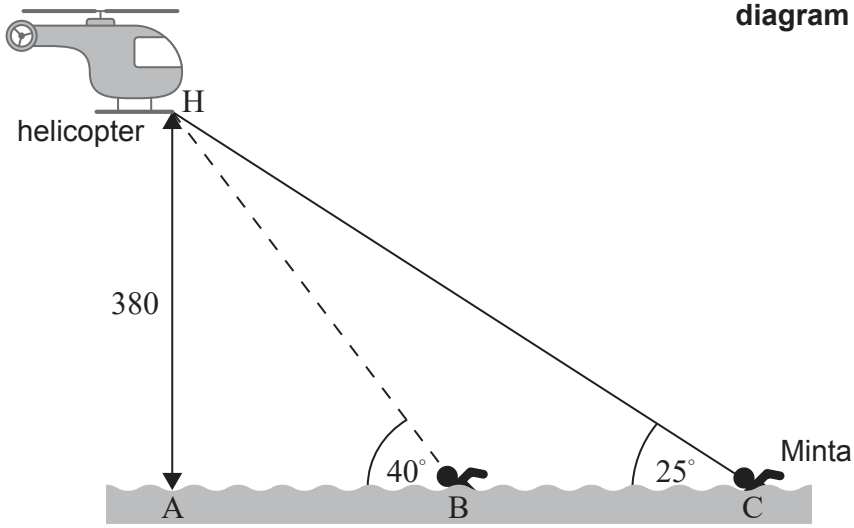
4. [Maximum mark: 7]

The diagram below shows a helicopter hovering at point H, 380 m vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.

diagram not to scale



Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of 25°. After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of 40°.

(a) Write down the size of the angle of depression from H to C. [1]

(b) Find the distance from A to C. [2]

(c) Find the distance from B to C. [3]

(d) Find Minta's speed, in metres per hour. [1]

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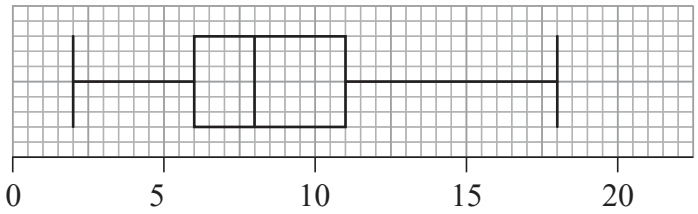
5. [Maximum mark: 5]

The number of sick days taken by each employee in a company during a year was recorded. The data was organized in a box and whisker diagram as shown below:



(a) For this data, write down

(i) the minimum number of sick days taken during the year.

(ii) the lower quartile.

(iii) the median. [3]

Paul claims that this box and whisker diagram can be used to infer that the percentage of employees who took fewer than six sick days is smaller than the percentage of employees who took more than eleven sick days.

(b) State whether Paul is correct. Justify your answer. [2]

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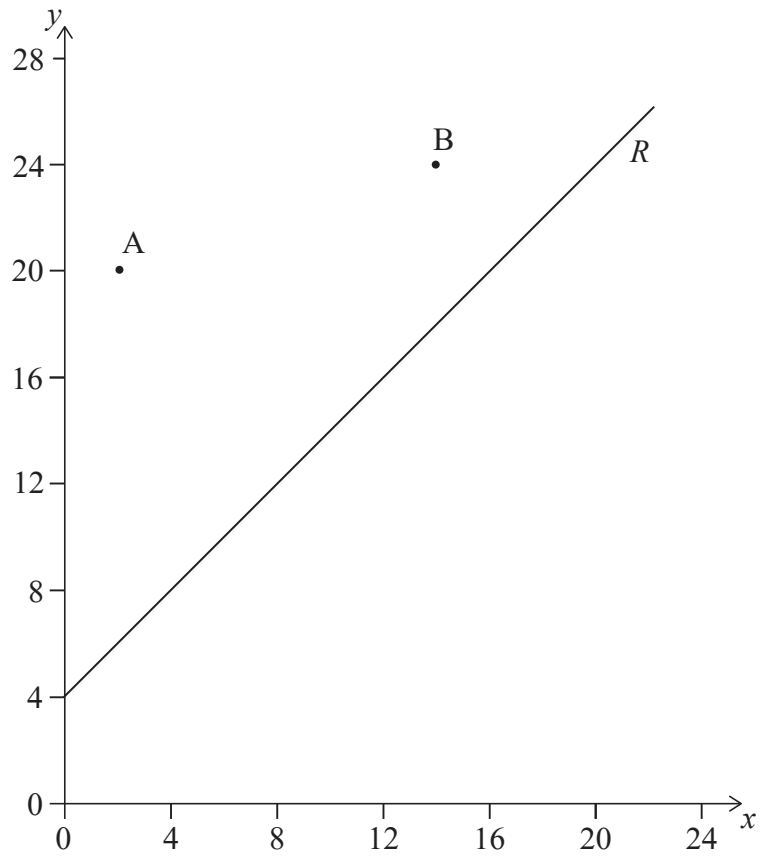
6. [Maximum mark: 7]

Two schools are represented by points $\mathrm{A}(2,\ 20)$ and $\mathrm{B}(14,\ 24)$ on the graph below. A road, represented by the line R with equation $-x + y = 4$, passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.



(a) Find the equation of the perpendicular bisector of $[\mathrm{AB}]$. Give your equation in the form $y = mx + c$. [5]

(b) Determine the coordinates of the point on R where the bus stop should be located. [2]

(This question continues on the following page)

(Question 6 continued)

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7. [Maximum mark: 5]

A function is defined by $f(x) = 2 - \frac{12}{x+5}$ for $-7 \leq x \leq 7$, $x \neq -5$.

(a) Find the range of f. [3]

(b) Find the value of $f^{-1}(0)$. [2]

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8. [Maximum mark: 6]

At Springfield University, the weights, in kg, of 10 chinchilla rabbits and 10 sable rabbits were recorded. The aim was to find out whether chinchilla rabbits are generally heavier than sable rabbits. The results obtained are summarized in the following table.

Weight of chinchilla rabbits, kg	4.9	4.2	4.1	4.4	4.3	4.6	4.0	4.7	4.5	4.4
Weight of sable rabbits, kg	4.2	4.1	4.1	4.2	4.5	4.4	4.5	3.9	4.2	4.0

A t-test is to be performed at the 5% significance level.

(a) Write down the null and alternative hypotheses. [2]

(b) Find the p-value for this test. [2]

(c) Write down the conclusion to the test. Give a reason for your answer. [2]

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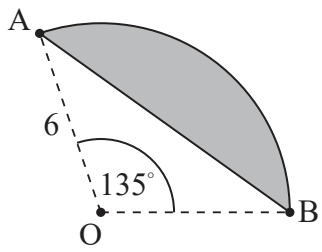
9. [Maximum mark: 7]

A garden includes a small lawn. The lawn is enclosed by an arc AB of a circle with centre O and radius $6\,\mathrm{m}$, such that $\mathrm{A\hat{O}B} = 135^\circ$. The straight border of the lawn is defined by chord $[\mathrm{AB}]$.

The lawn is shown as the shaded region in the following diagram.

diagram not to scale



(a) A footpath is to be laid around the curved side of the lawn. Find the length of the footpath. [3]

(b) Find the area of the lawn. [4]

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10. [Maximum mark: 6]

Tommaso and Pietro have each been given 1500 euro to save for college.

Pietro invests his money in an account that pays a nominal annual interest rate of 2.75%, **compounded half-yearly**.

(a) Calculate the amount Pietro will have in his account after 5 years. Give your answer correct to 2 decimal places. [3]

Tommaso wants to invest his money in an account such that his investment will increase to 1.5 times the initial amount in 5 years. Assume the account pays a nominal annual interest of $r\%$ **compounded quarterly**.

(b) Determine the value of r. [3]

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11. [Maximum mark: 6]

A newspaper vendor in Singapore is trying to predict how many copies of *The Straits Times* they will sell. The vendor forms a model to predict the number of copies sold each weekday. According to this model, they expect the same number of copies will be sold each day.

To test the model, they record the number of copies sold each weekday during a particular week. This data is shown in the table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of copies sold	74	97	91	86	112

A goodness of fit test at the 5% significance level is used on this data to determine whether the vendor's model is suitable.

The critical value for the test is 9.49 and the hypotheses are

H_0 : The data satisfies the model.
H_1 : The data does not satisfy the model.

(a) Find an estimate for how many copies the vendor expects to sell each day. [1]

(b) (i) Write down the degrees of freedom for this test.

(ii) Write down the conclusion to the test. Give a reason for your answer. [5]

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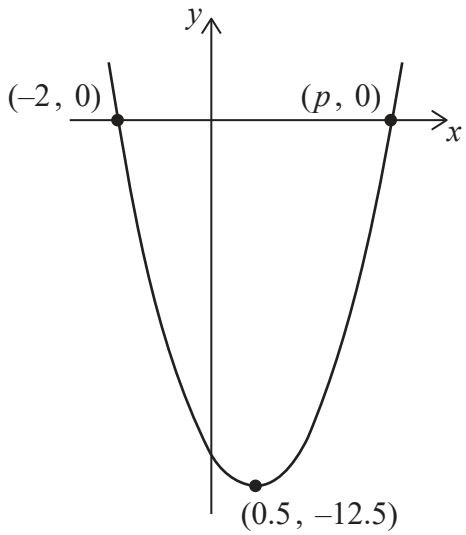
12. [Maximum mark: 7]

Consider the function $f(x) = ax^2 + bx + c$. The graph of $y = f(x)$ is shown in the diagram. The vertex of the graph has coordinates $(0.5, -12.5)$. The graph intersects the x-axis at two points, $(-2, 0)$ and $(p, 0)$.

diagram not to scale



(a) Find the value of p. [1]

(b) Find the value of

(i) a.

(ii) b.

(iii) c. [5]

(c) Write down the equation of the axis of symmetry of the graph. [1]

(This question continues on the following page)

(Question 12 continued)

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13. [Maximum mark: 7]

A company produces and sells electric cars. The company's profit, P, in thousands of dollars, changes based on the number of cars, x, they produce per month.

The rate of change of their profit from producing x electric cars is modelled by

$$\frac{\mathrm{d}P}{\mathrm{d}x} = -1.6x + 48\,,\ x \geq 0\,.$$

The company makes a profit of 260 (thousand dollars) when they produce 15 electric cars.

(a) Find an expression for P in terms of x. [5]

The company regularly increases the number of cars it produces.

(b) Describe how their profit changes if they increase production to over 30 cars per month and up to 50 cars per month. Justify your answer. [2]

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References: